

Relation between Vortex Pinning Energy and Anderson's Theorem

Nobuhiko Hayashi and Yusuke Kato*

Computer Center, Okayama University, Okayama 700-8530, Japan

**Department of Basic Science, University of Tokyo, Tokyo 153-8902, Japan*

We discuss the elementary vortex pinning in type-II superconductors in connection with the Anderson's theorem for nonmagnetic impurities. We address the following two issues. One is an enhancement of the vortex pinning energy in the unconventional superconductors. This enhancement comes from the pair-breaking effect of a nonmagnetic defect as the pinning center far away from the vortex core (i.e., the pair-breaking effect due to the non-applicability of the Anderson's theorem in the unconventional superconductors). The other is an effect of the chirality on the vortex pinning energy in a chiral p -wave superconductor. The vortex pinning energy depends on the chirality. This is related to the cancellation of the angular momentum between the vorticity and chirality in a chiral p -wave vortex core, resulting in local applicability of the Anderson's theorem (or local recovery of the Anderson's theorem) inside the vortex core.

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1. INTRODUCTION

Much attention has been focused on the vortex pinning in type-II superconductors. The vortex pinning governs the macroscopic magnetic properties of type-II superconductors such as the hysteresis of the magnetization and the critical current. The elementary vortex pinning potential is the interaction between a vortex and a single defect. A microscopic analysis of the elementary vortex pinning potential is necessary for understanding of the macroscopic vortex pinning problem in the superconductors.^{1, 2, 3, 4, 5}

The vortex pinning energy is given by the difference in the free energy

between the case when the vortex is located at the pinning center and the case when the vortex is far away from the pinning center. Naively, the elementary vortex pinning energy due to a defect with the volume V_i is expected to be given roughly by

$$\frac{1}{8\pi} p V_i H_c^2, \quad (1)$$

where H_c is the thermodynamic critical field and p is a numerical factor much smaller than unity. The superconductivity is destroyed at the defect and the energy gain of the superconducting condensation is lost just locally. When the position of the defect coincides with the vortex center, the loss of the condensation energy is avoided, because in the vortex core the superconducting order parameter is already depleted and therefore the vortex core could be regarded as a locally realized normal-state region. However, the above argument does not hold if once we take into account a nonlocal effect around the defect. Thuneberg *et al.*^{1,2} advanced the understanding of the mechanism of the elementary vortex pinning, taking into account of such a nonlocal effect that the defect scatters the quasiparticles around it as a scattering center. They calculated the vortex pinning energy for a vortex in isotropic *s*-wave superconductors (i.e., an isotropic *s*-wave vortex) and found that the vortex pinning energy is larger than Eq. (1) by a factor of ξ/d , where ξ is the coherence length and d the linear dimension of the defect.^{1,2} Thuneberg³ subsequently attacked the same problem by deriving the impurity (or defect) correction term of the Ginzburg-Landau (GL) theory for the isotropic *s*-wave superconductors. Friesen and Muzikar⁶ extended the GL method of Ref. 3 to superconductors with general pairing symmetry (including the unconventional superconductors). Kulić and Dolgov⁷ discussed the vortex pinning potential due to an anisotropic impurity in the unconventional superconductors. The present authors discussed, in Refs. 8, 9, a new pinning effect intrinsic to chiral *p*-wave superconductors.

With these backgrounds, we discuss, in this paper, a relation between the elementary vortex pinning potential and the Anderson's theorem for nonmagnetic impurities. Here, the Anderson's theorem means that nonmagnetic impurities (or nonmagnetic defects) do not affect the thermodynamic properties of a superconductor.^{10,11,12} We focus on the following two points. (i) The vortex pinning energy in the unconventional superconductors is enhanced, compared to that in the isotropic *s*-wave superconductors. This enhancement originates from the pair-breaking effect far away from the vortex core in the unconventional superconductors (i.e., the pair-breaking effect due to the well-known non-applicability of the Anderson's theorem in the unconventional superconductors¹²). Such a pair-breaking effect does not occur in the isotropic *s*-wave superconductors (owing to the applicability of the An-

Relation between Vortex Pinning and Anderson's Theorem

derson's theorem in homogeneous isotropic s -wave superconductors^{10,11,12}). The unconventional superconductivity¹² has been proposed for many superconductors such as high- T_c cuprates, organic conductors, and heavy-fermion compounds. Thus this issue becomes very important recently. (ii) We then consider an effect of the chirality on the vortex pinning energy in a chiral p -wave superconductor with an unconventional pairing^{13,14} $\mathbf{d} = \bar{\mathbf{z}}(\bar{k}_x \pm i\bar{k}_y)$, which is one of the unconventional superconductors. The vortex pinning energy depends on the chirality. This is related to local applicability of the Anderson's theorem (or local recovery of the Anderson's theorem) inside the vortex core of *chiral p -wave* superconductors.

In Sec. 2, we summarize the formulation of the quasiclassical theory of superconductivity used for the study of the vortex pinning. In Sec. 3, the Anderson's theorem for nonmagnetic impurities in the isotropic s -wave superconductors is described within the formalism of the quasiclassical theory. Even in the vortex states, the Anderson's theorem is applicable if the nonmagnetic defect is far away from the vortex core. By contrast, the nonmagnetic defect inside the vortex core affects the free energy of superconductors and yields the vortex pinning energy. In Sec. 4, we discuss the enhancement of the vortex pinning energy due to the pair-breaking effect of the nonmagnetic defect far away from the vortex core in the unconventional superconductors. In Sec. 5, we discuss the effect of the nonmagnetic defect inside the vortex core in the chiral p -wave superconductor. The inside of the chiral p -wave vortex core is similar to the homogeneous state of an isotropic s -wave superconductor. As an evidence, we show that the nonmagnetic defect inside the vortex core does not affect the free energy of the chiral p -wave superconductors. We regard this result as local applicability of the Anderson's theorem *inside the vortex core in the chiral p -wave superconductors*. The summary is given in Sec. 6.

2. FORMULATION

We adopt the quasiclassical theory of superconductivity^{15,16,17} to investigate the vortex pinning. We consider the quasiclassical Green function in the absence of the pinning,

$$\hat{g}_{\text{imt}}(i\omega_n, \mathbf{r}, \bar{\mathbf{k}}) = -i\pi \begin{pmatrix} g_{\text{imt}} & if_{\text{imt}} \\ -if_{\text{imt}}^\dagger & -g_{\text{imt}} \end{pmatrix}, \quad (2)$$

which is the solution of the Eilenberger equation,¹⁷

$$i\mathbf{v}_F(\bar{\mathbf{k}}) \cdot \nabla \hat{g}_{\text{imt}} + [i\omega_n \hat{\tau}_z - \hat{\Delta}, \hat{g}_{\text{imt}}] = 0, \quad (3)$$

where the superconducting order parameter is $\hat{\Delta}(\mathbf{r}, \bar{\mathbf{k}}) = [(\hat{\tau}_x + i\hat{\tau}_y)\Delta(\mathbf{r}, \bar{\mathbf{k}}) - (\hat{\tau}_x - i\hat{\tau}_y)\Delta^*(\mathbf{r}, \bar{\mathbf{k}})]/2$ and $\hat{\tau}_i$ are the Pauli matrices. $\mathbf{v}_F(\bar{\mathbf{k}})$ is the Fermi

N. Hayashi and Y. Kato

velocity, ω_n is the fermionic Matsubara frequency, and the commutator $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$. The Eilenberger equation is supplemented by the normalization condition $\hat{g}_{\text{imt}}(i\omega_n, \mathbf{r}, \bar{\mathbf{k}})^2 = -\pi^2 \hat{1}$.¹⁷ Since we consider, in this paper, an isolated single vortex in extreme type-II superconductors where the Ginzburg-Landau parameter $\kappa \gg 1$, the vector potential is neglected in Eq. (3). We use units in which $\hbar = k_B = 1$.

In this paper, the system is assumed to be a two dimensional conduction layer perpendicular to the magnetic field. The vector $\mathbf{r} = (r \cos \phi, r \sin \phi)$ is the center of mass coordinate. The unit vector $\bar{\mathbf{k}} = (\cos \theta, \sin \theta)$ represents the wave number of relative motion of the Cooper pairs. We assume a circular Fermi surface and $\mathbf{v}_F(\bar{\mathbf{k}}) = v_F \bar{\mathbf{k}} = (v_F \cos \theta, v_F \sin \theta)$.

The effect of the pinning is introduced to the quasiclassical theory of superconductivity as follows.^{1, 2, 18} The quasiclassical Green function \hat{g} in the presence of a point-like nonmagnetic defect situated at $\mathbf{r} = \mathbf{R}$ is obtained from the Eilenberger equation

$$i\mathbf{v}_F(\bar{\mathbf{k}}) \cdot \nabla \hat{g} + [i\omega_n \hat{\tau}_z - \hat{\Delta}, \hat{g}] = [\hat{t}, \hat{g}_{\text{imt}}] \delta(\mathbf{r}'), \quad (4)$$

and the t matrix due to the nonmagnetic defect (or nonmagnetic impurity)

$$\hat{t}(i\omega_n, \mathbf{r}') = \frac{v}{D} \left[\hat{1} + N_0 v \langle \hat{g}_{\text{imt}}(i\omega_n, \mathbf{r}', \bar{\mathbf{k}}) \rangle_\theta \right], \quad (5)$$

where $\mathbf{r}' = \mathbf{r} - \mathbf{R}$, the denominator $D = 1 + (\pi N_0 v)^2 [\langle g_{\text{imt}} \rangle_\theta^2 + \langle f_{\text{imt}} \rangle_\theta \langle f_{\text{imt}}^\dagger \rangle_\theta]$, the average over the Fermi surface $\langle \cdots \rangle_\theta = \int \cdots d\theta / 2\pi$, the normal-state density of states on the Fermi surface N_0 , and we assume the s -wave scattering v when obtaining Eq. (5). We define a parameter $\sin^2 \delta_0 = (\pi N_0 v)^2 / [1 + (\pi N_0 v)^2]$, which measures how strong the scattering potential of the nonmagnetic defect is.

The vortex pinning potential $\delta\Omega(R)$ ($R \equiv |\mathbf{R}|$), i.e., the difference in the free energy between the states with and without the nonmagnetic defect is, at the temperature T , given as^{1, 2, 18, 19}

$$\begin{aligned} \delta\Omega(R) &= \Omega_p(R) - \Omega_0(R) \\ &= N_0 T \int_0^1 d\lambda \sum_{\omega_n=-\infty}^{\infty} \int \frac{d\theta}{2\pi} \int d\mathbf{r} \text{Tr}[\delta\hat{g}_\lambda \hat{\Delta}_b], \end{aligned} \quad (6)$$

where $\Omega_p(R)$ is the free energy in the *presence* of the nonmagnetic defect, $\Omega_0(R)$ the free energy in the *absence* of the nonmagnetic defect, $\delta\hat{g}_\lambda = \hat{g} - \hat{g}_{\text{imt}}$ is evaluated at $\hat{\Delta} = \lambda \hat{\Delta}_b$, and $\hat{\Delta}_b$ is the order parameter in the absence of the nonmagnetic defect.

Relation between Vortex Pinning and Anderson's Theorem

3. ELEMENTARY PINNING POTENTIAL FOR ISOTROPIC S-WAVE SUPERCONDUCTORS

The elementary pinning potential for the isotropic *s*-wave superconductors has been microscopically calculated first by Thuneberg *et al.*^{1,2} on the basis of the quasiclassical theory of superconductivity. In this section, we summarize their results.

3.1. Homogeneous System

In the homogeneous system (e.g., far away from the vortex core), the matrix elements of the solution of the Eilenberger equation (3) are given as^{2,17,20}

$$g_{\text{imt}} = \frac{\omega_n}{\sqrt{\omega_n^2 + |\Delta|^2}}, \quad f_{\text{imt}} = \frac{\Delta}{\sqrt{\omega_n^2 + |\Delta|^2}}, \quad f_{\text{imt}}^\dagger = \frac{\Delta^*}{\sqrt{\omega_n^2 + |\Delta|^2}}, \quad (7)$$

where Δ is the spatially uniform order parameter in an isotropic *s*-wave superconductor. Because these have no $\bar{\mathbf{k}}$ - (or θ -) dependence, the Fermi-surface averages of them are

$$\langle g_{\text{imt}} \rangle_\theta = g_{\text{imt}}, \quad \langle f_{\text{imt}} \rangle_\theta = f_{\text{imt}}, \quad \langle f_{\text{imt}}^\dagger \rangle_\theta = f_{\text{imt}}^\dagger, \quad (8)$$

namely,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta = \hat{g}_{\text{imt}}. \quad (9)$$

Inserting this Eq. (9) into the impurity *t* matrix, Eq. (5), we obtain

$$\hat{t} = \frac{v}{D} \left[\hat{1} + N_0 v \hat{g}_{\text{imt}} \right], \quad (10)$$

and therefore the right hand side of Eq. (4) vanishes, namely,

$$[\hat{t}, \hat{g}_{\text{imt}}] = 0. \quad (11)$$

When $[\hat{t}, \hat{g}_{\text{imt}}] = 0$, the Eilenberger equation (4) in the presence of the nonmagnetic defect is identical to Eq. (3) (the equation in the absence of the nonmagnetic defect), namely, the impurity has no influence on the Green function and the free energy (thus, $\delta\Omega = 0$). This is consistent with the Anderson's theorem, which states that nonmagnetic impurities do not change the free energy of homogeneous isotropic *s*-wave superconductors.^{10,11,12} It is noted that the Anderson's theorem is described as the commutativity (Eq. (11)) between the impurity *t*-matrix, \hat{t} , and the intermediate green function,^{2,18} \hat{g}_{imt} , within the framework of the quasiclassical theory of superconductivity.

3.2. Nonmagnetic Impurity inside Vortex Core

In the spatially varying case, the Anderson's theorem does not apply. Therefore, the nonmagnetic defect within a vortex core affects the free energy of the system and has a contribution to the vortex pinning energy.

At the vortex center $\mathbf{r} = 0$, on the basis of an analysis of the so-called zero-core vortex model, the matrix elements of the solution of Eq. (3) are approximately given as²

$$g_{\text{imt}} = \frac{\sqrt{\omega_n^2 + |\tilde{\Delta}|^2}}{\omega_n}, \quad f_{\text{imt}} = \frac{-\tilde{\Delta}}{\omega_n}, \quad f_{\text{imt}}^\dagger = \frac{\tilde{\Delta}^*}{\omega_n}, \quad (12)$$

where $\tilde{\Delta} = \Delta(r \rightarrow \infty) \exp(i\theta)$. Here, the vortex with the order parameter $\Delta(\mathbf{r}) = \Delta(r) \exp(i\phi)$ is considered, and the amplitude of the order parameter $\Delta(r)$ is set to be constant (i.e., zero core) around the vortex, which is the approximation based on the zero-core vortex model.² When Eq. (12) is obtained, the quasiparticles which go through the origin $\mathbf{r} = 0$ are considered.² The position vectors of such quasiparticles are parallel to the Fermi velocity (i.e., $\mathbf{r} \parallel \mathbf{v}_F(\bar{\mathbf{k}})$), and therefore $\phi = \theta$ in $\tilde{\Delta}$ of Eq. (12).

The Fermi-surface averages of Eq. (12) are

$$\langle g_{\text{imt}} \rangle_\theta = g_{\text{imt}}, \quad \langle f_{\text{imt}} \rangle_\theta = 0, \quad \langle f_{\text{imt}}^\dagger \rangle_\theta = 0, \quad (13)$$

namely,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta \neq \hat{g}_{\text{imt}}. \quad (14)$$

Because of the phase factor $\exp(i\theta)$ of $\tilde{\Delta}$ in Eq. (12), the Fermi-surface averages of the anomalous Green functions, $\langle f_{\text{imt}} \rangle_\theta$ and $\langle f_{\text{imt}}^\dagger \rangle_\theta$, vanish in Eq. (13).

From Eqs. (5) and (14), it follows that the t matrix due to the nonmagnetic defect situated at the vortex center does not commute with the Green function \hat{g}_{imt} , namely $[\hat{t}, \hat{g}_{\text{imt}}] \neq 0$ in general. Therefore, a nonzero effect of the nonmagnetic defect appears in the right hand side of the Eilenberger equation (4) and the nonmagnetic defect situated at the vortex center affects the Green function and the free energy, $\delta\Omega(R = 0) = \Omega_p(R = 0) - \Omega_0(R = 0) < 0$. (R is the distance between the vortex center and the nonmagnetic defect.) The nonzero $\delta\Omega(R = 0)$ is an origin of the vortex pinning (see Fig. 1). In the isotropic s -wave superconductors (in which $\delta\Omega(R \rightarrow \infty) = 0$ as discussed in Sec. 3.1), it is the only origin of the vortex pinning.

According to Thuneberg *et al.*,^{1,2} an approximated analytical expression for the vortex pinning potential in the isotropic s -wave superconductors is given as

$$\delta\Omega(R = 0) = -2T \ln \cosh \left[\frac{|\tilde{\Delta}(T)| \sin \delta_0}{2T} \right], \quad (15)$$

Relation between Vortex Pinning and Anderson's Theorem

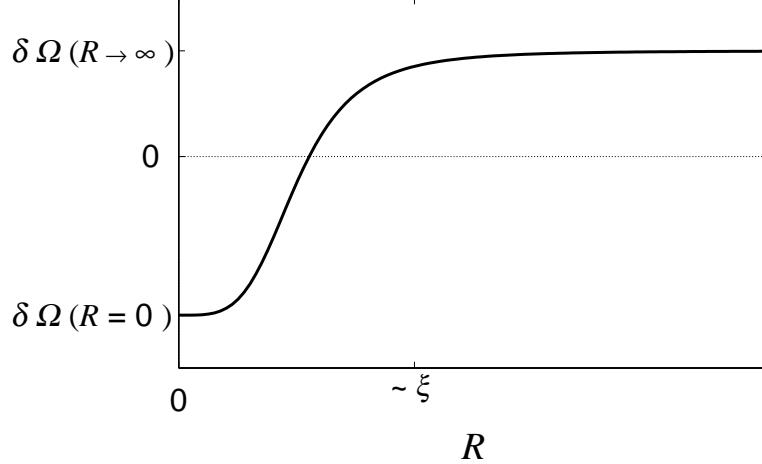


Fig. 1. Schematic figure of the vortex pinning potential $\delta\Omega(R)$ as a function of the distance R between the vortex center and the nonmagnetic defect. The recovery length of $\delta\Omega(R)$ is of the order of the coherence length ξ .

on the basis of the zero-core vortex model.² Here, $|\tilde{\Delta}(T)|$ has the temperature dependence of the BCS gap, $\Delta_{\text{BCS}}(T)$. From the quantitative viewpoint, however, it should be noted that at high temperatures Eq. (15) overestimates the magnitude $|\delta\Omega|$ at one order larger value as compared to a precise numerical result.^{1,2}

4. ENHANCEMENT OF PINNING ENERGY IN UNCONVENTIONAL SUPERCONDUCTORS

As seen in the preceding section, in the case of the isotropic s -wave superconductors, the nonmagnetic defect does not affect the vortex pinning potential $\delta\Omega$ far away from the vortex core (i.e., $\delta\Omega(R \rightarrow \infty) = 0$), because the Anderson's theorem is applicable to the homogeneous system in the isotropic s -wave superconductors. In the unconventional superconductors, on the other hand, the Anderson's theorem does not apply even in the homogeneous system.¹² This is because the order parameter Δ in Eq. (7) has a $\bar{\mathbf{k}}$ - (or θ -) dependence in such superconductors and hence $\langle \hat{g}_{\text{imt}} \rangle_\theta \neq \hat{g}_{\text{imt}}$ instead of Eq. (9). Therefore, the nonmagnetic defect affects the free energy and gives rise to the condensation energy loss, $\delta\Omega = \Omega_p - \Omega_0 > 0$, in the unconventional superconductors. Such a condensation energy loss far away from the vortex core, $\delta\Omega(R \rightarrow \infty)$, contributes to the depth of the vortex

pinning potential (see Fig. 1), namely, to the vortex pinning energy,^{18,6,7,8}

$$E_{\text{pin}} = \delta\Omega(R \rightarrow \infty) - \delta\Omega(R = 0). \quad (16)$$

While the vortex pinning energy is just $E_{\text{pin}} = -\delta\Omega(R = 0)$ in the isotropic *s*-wave superconductors, in the case of the unconventional superconductors the additional contribution from $\delta\Omega(R \rightarrow \infty)$ may enhance the vortex pinning energy E_{pin} . In what follows, from Eq. (6) we calculate $\delta\Omega(R \rightarrow \infty)$ for the chiral *p*-wave superconductor as an example of the unconventional superconductors.

For the chiral *p*-wave pairing state^{13,14} $\mathbf{d} = \bar{\mathbf{z}}(\bar{k}_x \pm i\bar{k}_y) = \bar{\mathbf{z}} \exp(\pm i\theta)$, we consider the condensation energy loss when putting a nonmagnetic defect on the homogeneous system. The spatially uniform order parameter in the homogeneous system is expressed as $\Delta_b(\bar{\mathbf{k}}) = \Delta_b(\theta) = \Delta_b^0 \exp(\pm i\theta)$. The matrix elements of the solution of Eq. (3) are given by

$$g_{\text{imt}} = \frac{\omega_n}{\sqrt{\omega_n^2 + |\Delta_b^0|^2}}, \quad f_{\text{imt}} = \frac{\Delta_b(\theta)}{\sqrt{\omega_n^2 + |\Delta_b^0|^2}}, \quad f_{\text{imt}}^\dagger = \frac{\Delta_b^*(\theta)}{\sqrt{\omega_n^2 + |\Delta_b^0|^2}}, \quad (17)$$

as in Eq. (7). The Fermi-surface averages of Eq. (17) are

$$\langle g_{\text{imt}} \rangle_\theta = g_{\text{imt}}, \quad \langle f_{\text{imt}} \rangle_\theta = 0, \quad \langle f_{\text{imt}}^\dagger \rangle_\theta = 0, \quad (18)$$

because $\langle \Delta_b(\theta) \rangle_\theta = 0$. Inserting these into Eq. (5), we obtain the commutator $[\hat{t}, \hat{g}_{\text{imt}}]$ as

$$[\hat{t}, \hat{g}_{\text{imt}}] = \frac{A}{2} \left[(\hat{\tau}_x + i\hat{\tau}_y) \Delta + (\hat{\tau}_x - i\hat{\tau}_y) \Delta^* \right], \quad (19)$$

where

$$A = \frac{-2i\omega_n \sin^2 \delta_0}{N_0(\omega_n^2 + |\Delta|^2 \cos^2 \delta_0)}, \quad (20)$$

and $\Delta \equiv \lambda \Delta_b(\theta)$.

We take a coordinate system with the origin at the nonmagnetic defect: $\mathbf{r} = s\mathbf{v}_F(\bar{\mathbf{k}})/v_F + b[\bar{\mathbf{z}} \times \mathbf{v}_F(\bar{\mathbf{k}})/v_F] = s\bar{\mathbf{k}} + b(\bar{\mathbf{z}} \times \bar{\mathbf{k}})$. In this coordinate system, $i\mathbf{v}_F(\bar{\mathbf{k}}) \cdot \nabla = iv_F d/ds$ and the Eilenberger equation (4) is written as

$$iv_F \frac{d}{ds} \hat{g} + [i\omega_n \hat{\tau}_z - \hat{\Delta}, \hat{g}] = [\hat{t}, \hat{g}_{\text{imt}}] \delta(s) \delta(b), \quad (21)$$

and $\delta\hat{g}_\lambda$ ($= \hat{g} - \hat{g}_{\text{imt}}$) in Eq. (6) follows the equation,

$$iv_F \frac{d}{ds} \delta\hat{g}_\lambda + [i\omega_n \hat{\tau}_z - \hat{\Delta}, \delta\hat{g}_\lambda] = [\hat{t}, \hat{g}_{\text{imt}}] \delta(s), \quad (22)$$

Relation between Vortex Pinning and Anderson's Theorem

where $\delta\hat{g}_\lambda = 0$ for $b \neq 0$.

Using the Fourier transformations, $\delta\hat{g}_\lambda(s) = \int dq \exp(iqs)\delta\hat{g}_\lambda(q)/2\pi$, $\delta(s) = \int dq \exp(iqs)/2\pi$, and a decomposition $\delta\hat{g}_\lambda(s) = g_1\hat{\tau}_x + g_2\hat{\tau}_y + g_3\hat{\tau}_z$, we can solve Eq. (22) into which Eq. (19) is inserted. From its solution $\delta\hat{g}_\lambda(q)$, we obtain

$$\begin{aligned}\int d\mathbf{r}\delta\hat{g}_\lambda &= \int ds\delta\hat{g}_\lambda(s) \\ &= \delta\hat{g}_\lambda(q=0) \\ &= \frac{A\omega_n[-i(\Delta - \Delta^*)\hat{\tau}_x + (\Delta + \Delta^*)\hat{\tau}_y + (4\omega_n^2 + 2|\Delta|^2)\hat{\tau}_z]}{4(\omega_n^2 + |\Delta|^2)},\end{aligned}\quad (23)$$

and therefore, from Eq. (6),

$$\begin{aligned}\delta\Omega &= N_0 T \int_0^1 d\lambda \sum_{\omega_n} \int \frac{d\theta}{2\pi} \frac{2\omega_n^2 \lambda |\Delta_b|^2 \sin^2 \delta_0}{N_0(\omega_n^2 + \lambda^2 |\Delta_b|^2 \cos^2 \delta_0)(\omega_n^2 + \lambda^2 |\Delta_b|^2)} \\ &= 2T \ln \left[\frac{\cosh(|\Delta_b^0(T)|/2T)}{\cosh(|\Delta_b^0(T)| \cos \delta_0/2T)} \right].\end{aligned}\quad (24)$$

This is the condensation energy loss due to the pair-breaking effect, $\delta\Omega(R \rightarrow \infty)$, for the chiral p -wave superconductor. Here, $|\Delta_b^0(T)|$ has the temperature dependence of the BCS gap, $\Delta_{\text{BCS}}(T)$.

At high temperatures, the value $\delta\Omega(R \rightarrow \infty)$ of Eq. (24) becomes the same order as the magnitude $|\delta\Omega(R=0)|$ of Eq. (15). Therefore, it appears that at high temperatures the vortex pinning energy in the unconventional superconductors, $E_{\text{pin}} (= \delta\Omega(R \rightarrow \infty) + |\delta\Omega(R=0)|)$, is enhanced about twice compared to the isotropic s -wave superconductor in which $E_{\text{pin}} = |\delta\Omega(R=0)|$, owing to the additional contribution from $\delta\Omega(R \rightarrow \infty)$. However, since the approximated expression, Eq. (15), overestimates the magnitude $|\delta\Omega(R=0)|$ at one order larger value at high temperatures,^{1,2} we may expect that the value of $\delta\Omega(R \rightarrow \infty)$ is two or more times larger than an actual value of $|\delta\Omega(R=0)|$. Indeed we have found, using a numerical calculation, that at high temperatures the vortex pinning energy E_{pin} in the case of the chiral p -wave superconductor⁹ (and a d -wave superconductor⁸) is about 10 times larger than that of the isotropic s -wave pairing case, owing to $\delta\Omega(R \rightarrow \infty) \sim 10 \times |\delta\Omega(R=0)|$.^{8,9} This enhancement of the vortex pinning energy E_{pin} is due to the pair-breaking effect of the nonmagnetic defect far away from the vortex center and therefore such an enhancement is a common feature of the unconventional superconductors.

5. ANDERSON'S THEOREM INSIDE VORTEX CORE IN CHIRAL SUPERCONDUCTOR

As described in Sec. 3, the Anderson's theorem does not apply to spatially varying system (e.g., the vortex core) and therefore the nonmagnetic defect inside the vortex core affects the free energy $\delta\Omega(R = 0)$ and acts as a pinning center. In general, it holds also for the unconventional superconductors. However, an exception exists. In this section, we discuss novel applicability of the Anderson's theorem inside the vortex core in the chiral p -wave superconductors; for those superconductors, the nonmagnetic defect inside a vortex core does not contribute to the free energy and, as a result, the vortex pinning potential at the vortex center turns out to be $\delta\Omega(R = 0) = 0$. This phenomenon originates from a quantum effect, i.e., a cancellation of the phase factors of the superconducting order parameter.

For the chiral p -wave pairing state^{13,14} $\mathbf{d} = \bar{\mathbf{z}}(\bar{k}_x \pm i\bar{k}_y) = \bar{\mathbf{z}} \exp(\pm i\theta)$, it is known that the order parameter around a single vortex $\Delta(\mathbf{r}, \bar{\mathbf{k}}) [\equiv \Delta(r, \phi; \theta)]$ has two possible forms depending on whether the chirality and vorticity are parallel or antiparallel each other.^{21,22,23} One form is

$$\Delta^{+-}(r, \phi; \theta) = \Delta_+(r) e^{i(\theta-\phi)} + \Delta_-(r) e^{i(-\theta+\phi)}, \quad (25)$$

where the chirality and vorticity are antiparallel [$p(+-)$ -case]. The other is

$$\Delta^{++}(r, \phi; \theta) = \Delta_+(r) e^{i(\theta+\phi)} + \Delta_-(r) e^{i(-\theta+3\phi)}, \quad (26)$$

where the chirality and vorticity are parallel [$p(++)$ -case]. Here, the vortex center is situated at $\mathbf{r} = 0$, the dominant component $\Delta_+(r \rightarrow \infty) = \Delta_{\text{BCS}}(T)$, and the induced one $\Delta_-(r \rightarrow \infty) = 0$. Because of axisymmetry of the system, we can take $\Delta_{\pm}(r)$ to be real.

From the quasiclassical viewpoint, the quasiparticles inside a vortex core run along straight lines called as quasiparticle paths.^{24,25,26} We consider the quasiparticle paths which go through the origin $\mathbf{r} = 0$. On those paths, the position vector is parallel to the direction of the quasiparticle path (i.e., $\mathbf{r} \parallel \bar{\mathbf{k}}$), and therefore $\phi = \theta, \theta + \pi$. In this situation ($\phi = \theta$), from Eqs. (25) and (26), the order parameter on the path is

$$\Delta^{+-}(r, \phi = \theta; \theta) = \Delta_+(r) + \Delta_-(r) \quad (27)$$

in $p(+-)$ -case, and

$$\Delta^{++}(r, \phi = \theta; \theta) = [\Delta_+(r) + \Delta_-(r)] e^{2i\theta} \quad (28)$$

in $p(++)$ -case. The cancellation between the phase factors, $\exp(i\theta)$ due to the chirality of the Cooper pair and $\exp(i\phi)$ due to the vorticity of the vortex, occurs in Eq. (27) and not in Eq. (28).

Relation between Vortex Pinning and Anderson's Theorem

On the basis of the zero-core vortex model,² as in Eq. (12) the matrix elements of \hat{g}_{imt} at the vortex center are approximately obtained as

$$g_{\text{imt}} = \frac{\sqrt{\omega_n^2 + |\tilde{\Delta}|^2}}{\omega_n}, \quad f_{\text{imt}} = \frac{-\tilde{\Delta}}{\omega_n}, \quad f_{\text{imt}}^\dagger = \frac{\tilde{\Delta}^*}{\omega_n}, \quad (29)$$

where $\tilde{\Delta} = \Delta^{+\pm}(r \rightarrow \infty, \phi = \theta; \theta)$. Inserting the order parameter of Eq. (27) into Eq. (29), we obtain the Green function integrated over the Fermi surface as, in $p(+-)$ -case,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta = \hat{g}_{\text{imt}}, \quad \langle f_{\text{imt}} \rangle_\theta = f_{\text{imt}}, \quad \langle f_{\text{imt}}^\dagger \rangle_\theta = f_{\text{imt}}^\dagger, \quad (30)$$

namely,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta = \hat{g}_{\text{imt}}, \quad (31)$$

because of the absence of any phase factors in Eq. (27), i.e., because the cancellation between the chirality factor $\exp(i\theta)$ and the vorticity factor $\exp(-i\phi)$ occurs and then $\langle \Delta^{+-} \rangle_\theta = \Delta^{+-}$. We obtain $[\hat{t}, \hat{g}_{\text{imt}}] = 0$ from Eqs. (5) and (31). Therefore, following the same discussion as in Sec. 3.1, we conclude that in this $p(+-)$ -case the nonmagnetic defect at the vortex center does not affect the free energy and $\delta\Omega(R=0) = 0$. This situation is the same as that of the homogeneous system in the isotropic s -wave superconductors (see Sec. 3.1). That is, the Anderson's theorem is applicable (or is recovered) locally at the vortex center in $p(+-)$ -case when the chirality is antiparallel to the vorticity.

On the other hand, in $p(++)$ -case,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta = \hat{g}_{\text{imt}}, \quad \langle f_{\text{imt}} \rangle_\theta = 0, \quad \langle f_{\text{imt}}^\dagger \rangle_\theta = 0, \quad (32)$$

namely,

$$\langle \hat{g}_{\text{imt}} \rangle_\theta \neq \hat{g}_{\text{imt}}, \quad (33)$$

because the phase factor $\exp(2i\theta)$ is contained in Eq. (28) and then $\langle \Delta^{++} \rangle_\theta = 0$. Therefore, $[\hat{t}, \hat{g}_{\text{imt}}] \neq 0$ from Eqs. (5) and (33), and $\delta\Omega(0) \neq 0$ in this $p(++)$ -case when the sense of the chirality is the same as that of the vorticity. This is the same as the impurity effect inside the vortex core discussed in Sec. 3.2.

The above analysis in this section is based on the zero-core vortex model, i.e., on the non-self-consistent (constant) amplitude of the order parameter, but the phase of the order parameter, which is important for the above results, is correctly taken into account by that zero-core vortex model. Therefore, the essential physics is captured by the above analysis. We have certainly confirmed it by a numerical calculation of $\delta\Omega(R)$ based on self-consistently obtained order parameter.⁹ Related phenomena have been discussed in Refs. 27, 28, 29, 30 in different contexts.

Table 1

The Anderson's theorem is applicable (○) or *not* applicable (×) at the vortex center ($R = 0$) and far away from the vortex core ($R \rightarrow \infty$) in certain superconductors. If applicable (○), the vortex pinning potential $\delta\Omega$ becomes zero there. Refer also to Fig. 1.

| Superconductors | $R = 0$ | $R \rightarrow \infty$ |
|---------------------------------------------|---------|------------------------|
| <i>s</i> -wave | × | ○ |
| unconventional [general] | × | × |
| chiral <i>p</i> -wave [<i>p</i> (+−)-case] | ○ | × |

6. SUMMARY

We have discussed the relation between the vortex pinning energy and the Anderson's theorem, which is summarized in Table 1. The vortex pinning energy, E_{pin} (Eq. (16)), is given by the difference in the free energy between the case when vortex is far away from the nonmagnetic defect $\delta\Omega(R \rightarrow \infty)$ and the case when the vortex is located at the nonmagnetic defect $\delta\Omega(R = 0)$. The nonmagnetic defect far away from the vortex core can reduce locally the condensation energy of superconductors. On the other hand, the nonmagnetic defect within the vortex core can yield the energy gain to superconductors (through the scattering of quasiparticles in zero energy bound state). These two factors, $\delta\Omega(R \rightarrow \infty)$ and $\delta\Omega(R = 0)$, determine the magnitude of the vortex pinning energy E_{pin} for superconductors (Fig. 1). However, one of these factors happens to vanish in the following two cases. (1) In the isotropic *s*-wave superconductors, the nonmagnetic defect far away from the vortex core does not change the free energy, i.e., $\delta\Omega(R \rightarrow \infty) = 0$; this is well known as the Anderson's theorem.^{10,11,12} (2) The nonmagnetic defect inside the chiral *p*-wave vortex core, does not change the free energy and hence does not yield the energy gain to superconductors, i.e., $\delta\Omega(R = 0) = 0$, if the total angular momentum or (equivalently) the sum of the vorticity and the chirality is zero (*p*(+−)-case in Sec. 5). The vanishing angular momentum (or the resulting absence of the phase factors in the order parameter of Eq. (27)) makes the chiral *p*-wave vortex core similar to bulk *s*-wave superconductors. Therefore, the absence of the impurity effect in the chiral *p*-wave vortex core can be regarded as a consequence of the Anderson's theorem. This is the local applicability of the Anderson's theorem (or the local recovery of the Anderson's theorem) inside the chiral *p*-wave vortex core.

If we compare the isotropic *s*-wave superconductor with the (generic) unconventional superconductors in Table 1, the vortex pinning energy of the latter is larger than that of the former by the loss of the condensation

Relation between Vortex Pinning and Anderson's Theorem

energy at $R = \infty$. In high- T_c cuprates, this enhancement might be one of the reasons why small defects such as oxygen vacancies^{5,31} and Zn atoms³² are efficient pinning centers, because the high- T_c cuprates are believed to be d -wave superconductors (i.e., the unconventional superconductors). On the other hand, the chirality dependence of the vortex pinning is expected to be experimentally observed in future, for example, in a superconducting material Sr_2RuO_4 .^{13,14,33}

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N. Hayashi and Y. Kato

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